

# Geometrical Algorithms for Real Time Sound Rendering Using Intelligent Prioritization

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## Abstract

*Geometrical algorithms have been the main subject of research in the field of real time sound rendering. These algorithms are variants of the image source and ray tracing algorithms, enhanced with improvements that speed up substantially their performance. The fundamental concepts behind the improvements achieved up to now was the reduction of the processed information and the acceleration of the actual processing. In this paper, we show how altering the traversal method affects significantly the algorithm's performance. These optimizations alter its behavior, providing better results for real time purposes. We separate the techniques into three major categories and we propose a stochastic Monte Carlo algorithm which involves optimizations based on prioritization.*

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## 1. Introduction

Sound is an important component of an immersive 3D virtual environment as serves significant functions in such environments such as providing the sense of location and orientation, creating emotions and enhancing user immersion. Hence, delivering realistic sound in 3D virtual environments is a non-negligible task for such applications. As a result, a significant amount of research is devoted in delivering high quality audio in virtual reality and games applications. One of the areas of research in this field is the realistic simulation of sound propagation in 3D space for interactive applications.

In this paper, we present a method for improving the performance of geometrical acoustics algorithms used in sound rendering based on the concept of prioritization. Essentially, we propose improvements to a widely used tracing algorithm which alter its behavior and improve its performance for real time purposes. Our improvements are based on the concept of decomposing the tracing problem into a tree traversal problem and prioritizing the traversal of the tree in a way that more valid specular reflections are detected over the same amount of time. Our prioritization technique is based on automatically adjusting tracing termination criteria during runtime instead of explicitly setting them beforehand. We validate the algorithm based on paths detected, relative sound pressure and reverberation time calculated over a fixed amount of time. We compare the algorithm with other widely used techniques. Experimental executions of the algorithm indicate that this technique provides perceptually important improvements on almost all cases.

### 1.1. Related Work

We classify the related work based on the way they handle processing to improve performance. The categories we propose are processing reduction algorithms, processing acceleration algorithms and processing prioritization algorithms.

Tree pruning algorithms are those algorithmic techniques that result to the reduction of the size of the tree, therefore improving execution times substantially. Most of the advances in sound propagation calculations fall under this category. Most notable techniques are visibility tracing [Mec02], beam tracing [FTC\*04], frustum tracing [CLT\*08] and ray tracing [KSS68]. All these techniques share the same concept, that of tracing geometrical primitives through the 3D environment and detecting which geometrical objects are visible from other objects. By neglecting non visible objects from further processing, the tracing results to a pruned tree, which includes only nodes which are geometrically visible from their parent nodes.

Beam and frustum tracing techniques are based on the image source method introduced by Alen and Berkley [AB79] for rectangular rooms and extended by Borish for arbitrary polyhedral [Bor84]. Since then, several improvements were suggested, like Vörlander's hybrid ray tracing/image source implementation, Mechel's validity criteria [Mec02] and Schröder's binary space partitioning [SL06].

Beam tracing is a method that uses beams to accurately prune the tree of possible image sources, by only considering the visible leaves of each parent node. Beam tracing is currently considered as the fastest commonly used geometric room acoustics modeling technique [LSL09]. Recent developments in this area include the

development of priority based beam tracing [MF00], bidirectional beam tracing, amortized beam tracing [FMC99], beam tracing using precomputed visibility diagrams [AFST04], beam tracing using binary space partitioning [LSLS09], multi-threaded beam tracing [SM13].

Ray tracing is another popular method used in sound rendering. The most recent developments in ray tracing for sound rendering include the development of hybrid algorithms combining ray tracing with frustum tracing and methods for artificial reverb estimation [TCAM09], algorithms for the calculation of sound diffraction [OOK12], ray tracing using multi-view ray casting [TCM\*12], ray tracing using acceleration structures [DDJ\*12] and ray tracing for higher order diffractions and diffused reflections [SMM14].

Frustum tracing is a method that combines ray and beam tracing concepts. Lauterbach et al. [LCM07] presented the first frustum tracing algorithms applied in sound propagation. Chandak et al. [CLT\*08] proposed an improved version of frustum tracing called adaptive frustum tracing which adaptively refines the quadtree in order to perform accurate intersection computations with the primitives in the scene and generate new frusta. Taylor et al. [TCR\*09] use frustum tracing to calculate sound diffraction in complex environments.

Precomputation is used for the calculation of perceptual characteristics of the environment such the perceptual importance of sound sources, for the reduction of the environment's complexity and the calculation of transfer factors. Tsingos [TGD04] presents a method for the precomputation and perceptual assessment of spectral features of the input signals and also for precomputing geometry based reverberation effects [Tsi09]. Foale et al. [FV07] use precomputations for caching offline sound propagation calculations based on the portal subdivision method. Siltanen et al. [S\*10] as well as Drechsler [Dre14] use precomputation for the reduction of the model's geometrical complexity. Raguvanshi et al. precompute impulse responses for complex scenes and interpolate in real time for moving sources and receivers [?]. Antani et al. [ACTM12a] precompute the acoustic transfer operators using a technique similar to precomputed light transport. Stavrakis et al. precompute transport operators between coupled spaces connected by a portal to compute reverberation decay envelopes at interactive rates [STC08] and Mehra et al. precompute transfer operators based on equivalent sources [MMAR12].

Traversal prioritization is the concept of intelligently prioritizing the traversal of the tree, in such a way that the ratio of valid nodes over the total nodes is increased. This way more valid nodes are discovered during the same amount of time. Work on this field has been presented by Min and Funkhouser [MF00] and Charalampous and Michael [CM14b]. Koutsouris et al. [KBJJ13] also apply termination criteria based on the radiation densities of the walls.

## 2. Algorithm Prioritization

As we described in section 2, geometrical algorithms eventually detect a number of sound paths which are then used for the calculation of the impulse response. Most of the execution time is spent in detecting valid sound paths from a larger set of candidate sound

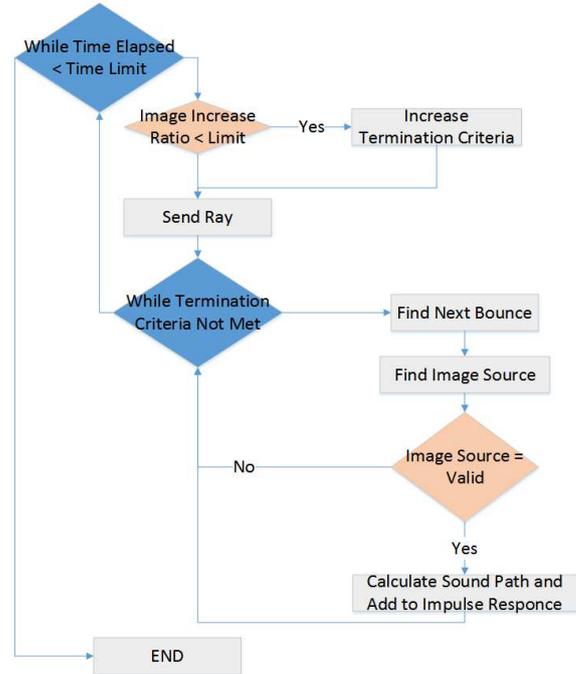


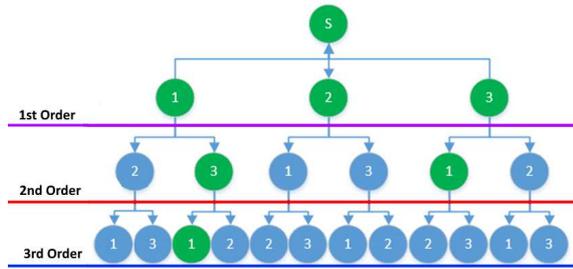
Figure 1: Algorithm flowchart.

paths. The set of candidate sound paths is usually orders of magnitude larger than the set of the valid ones. For example, Vörländer mentions that for the simple image source method for a rectangular room, the ratio between the number valid images and the total number of images is  $10^{12}$  [Vor89]. This indicates that most of the processing time goes wasted into evaluating non valid sound paths. At the same time, it is known that algorithms for real time sound propagation have strict time and resources constraints, thus it is practically impossible to evaluate all the candidates. An immediate hypothesis could be that if the processing of the superset of candidate sound paths was prioritized in an intelligent way, more valid paths could be detected at the earlier part of the path detection process rather than the latter. This could give a great benefit in terms of calculated results to algorithms designed for real time execution as more sound paths would be calculated within the same time frame. This paper investigates the benefits of applying prioritization techniques on algorithms widely used for real time rendering.

## 3. Prioritized Monte Carlo Algorithm

### 3.1. The Algorithm

Our proposed algorithm is based on a hybrid image source/ray tracing method [Vor89] which is extended using prioritization rules. Vörländer's method presented above improves significantly the performance of image source algorithm but when it comes to real time sound rendering, it has an important drawback. Vörländer's tracing process terminates when a) the ray intersects the receiver sphere or b) when it reaches a certain energy level or c) a predefined traveling distance is covered. In room-like enclosures, for which this algorithm was designed, this termination criterion works



**Figure 2: Tree Depth Termination Criterion.** Each time the termination criterion for the maximum order is increased, the depth of the searchable tree is increased by one.

well, because after some bounces on the wall, the ray eventually intersects with the receiver. But these criteria might not work well in other types of environments, indoor configurations with many rooms and outdoor configurations. The reason is because rays will be probably shot in directions from where it is not easy to return to the receiver and until they meet the termination criteria and the tracing is interrupted, a lot of computation time is wasted. Most modern ray tracing implementations use one of the following termination criteria a) a limit in sound reflections allowed per path, meaning that the propagation of a sound ray is terminated after a certain number of bounces has occurred and the receiver has not been reached [TCAM09] [TCM\*12] [SMM14] b) a minimum energy criterion where the ray propagation is terminated after its energy falls under a certain level [SS15] [RKM07] c) a maximum distance criterion where the ray propagation is terminated after the ray surpasses a predefined traveling distance [DDJ\*12]. The termination criteria are usually set arbitrarily, e.g. ten orders of sound reflections or a maximum distance of 1000, without any further discussion or based on a guessed perceptual importance e.g. sound paths that lose 60 dB are probably not affecting significantly the sound field.

We extend the above method by overcoming the issue of arbitrary set termination criteria using intelligent adjustment during runtime. A high level graphical description of the algorithm can be seen in Fig. 1. The criteria adjustment takes place during the tracing procedure and it is independent from user preferences.

### 3.2. Termination Criteria

Our algorithm uses the following three criteria 1) Tree depth (reflection order) 2) Parent validity 3) Sound pressure. In the following paragraphs we explain in detail each termination criterion and we also explain the procedure of adjusting the termination criteria during runtime.

#### 3.2.1. Tree Depth

There are two ways in which the image source order is important in an image source algorithm. Firstly, lower order images are usually stronger than higher order images since they are closer to the receiver and reflect fewer times, therefore losing less energy. Then, it is generally observed that each level of the image source tree has a

Box		Delta	
Sigma		Gamma	
Zeta		Epsilon	
Tau		Pi	
Ro		Theta	
Sigma LS		Gamma LS	
Zeta LS		Epsilon LS	
Tau LS		Pi LS	
Ro LS		Theta LS	

**Table 1: Models used to investigate termination criteria.** Source denoted with a red dodecahedron and receiver with a microphone.

lower density of valid image sources to total image sources than the previous levels [Vor89] [Mec02] [Mec12]. This can be expressed by the following relationship.

$$P(V_o) < P(V_{o-1}) \quad (1)$$

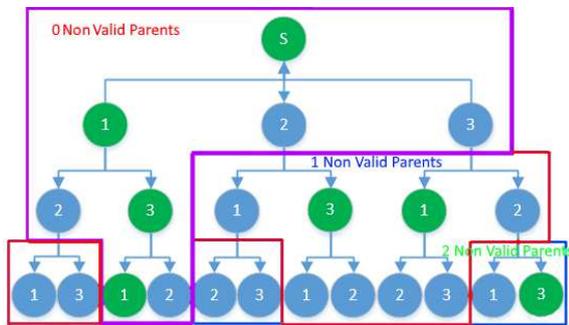
Where  $V$  indicates that an image source is valid and  $o$  the image source order. The above expression can be phrased as **the probability of an image of order  $o$  to be valid is less than the probability of an image of order  $o-1$ .**

Since this assumption cannot be easily proved in a mathematical way, we ran a series of experiments to strengthen our claim. We ran an improved version of the image source algorithm [Mec02] on eighteen different models (Table 1). We used ten different shapes for our models, a box and nine letter shaped rooms, in order to ensure adequate variety in geometrical settings. Then, for eight of the models that feature an occluded area, we used two different source-receiver configurations, one with line of sight between source and receiver (LS) and one without. After that, for each case we recorded the percentage of valid to total image sources for each order. The results are displayed in Table 2.

The first termination criterion is extracted by observing Table 2. We can observe that in the vast majority of cases, the valid image source density is higher in lower reflection orders. This means that if more rays explore the higher levels of the tree rather than the lower, there is an increased possibility of detecting valid image sources. In a tracing algorithm, where the rays continue their propagation through the environment until the termination order or the maximum distance criterion is met, therefore examining higher order images, the probability to generate a valid source decreases. Taking into account also the fact that lower ordered images con-

Geometry	1	2	3	4	5	6
Delta	100 %	52.0 %	25.7 %	13.2 %	6.7 %	3.4 %
Box	100 %	44.4 %	14 %	6.3 %	1.9 %	1.0 %
Gamma	- %	- %	0.28 %	0.16 %	0.1 %	0.03 %
Sigma	- %	0.7 %	0.07 %	0.04 %	0.005 %	0.001 %
Zeta	- %	0.8 %	0.4 %	0.1 %	0.04 %	0.01 %
Epsilon	- %	0.7 %	0.1 %	0.05 %	0.01 %	0.002 %
Tau	- %	2.2 %	0.9 %	0.3 %	0.1 %	0.02 %
Pi	10 %	0.0 %	0.5 %	0.05 %	0.07 %	0.007 %
Ro	8.4 %	4.3 %	1.2 %	0.3 %	0.05 %	0.02 %
Theta	- %	0.9 %	0.2 %	0.06 %	0.02 %	0.004 %
Sigma LS	28.6 %	7.75 %	0.4 %	0.2 %	0.02 %	0.005 %
Gamma LS	87.5 %	17.5 %	6.2 %	1.3 %	0.4 %	0.09 %
Zeta LS	50.0 %	9.0 %	1.7 %	0.4 %	0.1 %	0.02 %
Epsilon LS	35.7 %	8.33 %	1.3 %	0.3 %	0.04 %	0.008 %
Tau LS	60.0 %	14.2 %	2.1 %	0.6 %	0.1 %	0.03 %
Pi LS	50.0 %	14.3 %	2.9 %	0.8 %	0.2 %	0.04 %
Ro LS	45.5 %	11.0 %	1.6 %	0.4 %	0.07 %	0.02 %
Theta LS	50 %	9.3 %	1.2 %	0.3 %	0.04 %	0.009 %

**Table 2:** Percentage of valid images for each tree level as related with reflection orders.



**Figure 3:** Parent Validity Termination Criterion.

tribute more to the sound field, as explained above, we start with a low maximum order termination criterion and by progressively adjusting it we give priority to the higher parts of the tree at the beginning of the execution.

### 3.2.2. Parent Validity

The parent validity termination criterion assumes that the probability of an image source having a valid parent image source is higher than the average density of valid sources in the parent level. This can be expressed by the following relationship.

$$P(V|PAR_V) > P(V|PAR_{NV}) \quad (2)$$

Where  $V$  indicates that an image source is valid,  $PAR_V$  a valid parent source and  $PAR_{NV}$  a non valid parent source. The above expression can be phrased as **The probability of an image with a valid parent source to be valid is higher than the probability of an image with a non valid parent source to be valid.**

In a similar way to the first termination criterion, we ran a series of experiments to strengthen this claim. We ran the improved image source algorithm on the same room like enclosures like for the first termination criterion. Then we recorded the percentage of valid children for parent images that are valid and the percentage of

Geometry	% VP children that are valid	% NVP children that are valid
Box	18.6 %	1.0 %
Delta	35.4 %	4.4 %
Gamma	8.2 %	0.04 %
Sigma	0.6 %	0.002 %
Zeta	2.9 %	0.02 %
Epsilon	4.3 %	0.003 %
Tau	1.8 %	0.04 %
Pi	1.2 %	0.02 %
Ro	4.9 %	0.02 %
Theta	4.7 %	0.005 %
Sigma LS	3.3 %	0.01 %
Gamma LS	8.5 %	0.18 %
Zeta LS	5.8 %	0.03 %
Epsilon LS	4.5 %	0.02 %
Tau LS	1.1 %	0.06 %
Pi LS	6.6 %	0.08 %
Ro LS	5.7 %	0.03 %
Theta LS	5.1 %	0.01 %

**Table 3:** Percentage of children of valid parents that are valid(left) and of children of non valid parents that are valid(right) up to the 6th order of reflection.

valid children for parent images that are not valid. The results are displayed in Table 3.

Based on the results of this experiment, our second termination criterion is related with the fact that it seems more probable that a valid image source has a valid parent source than a non valid one. Based on this, rays reaching a tree node with a non valid image source have less probabilities, to reach a valid child node than rays that reach a valid tree node. Therefore, we terminate a ray propagation as soon as a maximum number of non valid parent images has been reached within the tree path traversed while progressively adjusting the maximum number of non valid images criterion giving priority to the images that have more valid parents at the beginning of the execution.

### 3.2.3. Sound Pressure Attenuation

An improvement to previous studies [CM16] is the addition of a third termination criterion. A limitation of using only the first two termination criteria is that the algorithm becomes agnostic when it comes to the specific materials composing the 3D geometry. By only evaluating the maximum order and the parent validity criterion the algorithm will behave in a similar way irrespective of the sound absorption of each surface. For this reason we introduce a third termination criterion, which is the maximum sound pressure attenuation. Therefore, the tracing of a ray is interrupted whenever the pressure falls under a specific threshold. The relative sound pressure level termination threshold is initialized at a level equivalent to attenuation due to distance at 1000 meters. In each readjustment of the termination criteria, the minimum sound pressure threshold is decreased by adding to the distance the average distance between bounces up to that moment.

### 3.2.4. Termination Criteria Run-Time Adjustment

The major difference of our algorithm compared to traditional tracing implementations is the adjustment of its termination criteria during runtime. In previous work [CM16] the termination criteria were adjusted whenever the number of consecutive failed rays (rays that failed to produce a valid image source) was surpassing the

number of surfaces in the model. This was highlighted as one of the weaknesses of the approach due to the arbitrariness of the method and was indicated as a subject of future research. In the current work, we change the adjustment method using a less arbitrary approach. Our current method is based on comparing the consecutive failed rays to the ratio of the number of all evaluated images over the number of all valid images detected until the time of the comparison. If the number of consecutive failed rays surpasses the ratio then this is an indication that the tracing has reached a saturation point and the termination criteria are increased.

As outlined above, we increase the maximum order criterion and the maximum non valid parent images (MaxNVPI) criterion by one each time the consecutive failed images surpass the threshold value. We also decrease the minimum pressure based on the average distance between bounces.

#### 4. Criteria for Evaluation of Algorithmic Performance for Real Time Sound Rendering

In the following paragraphs we briefly describe the criteria chosen to evaluate the performance of the tested algorithms.

##### 4.1. Number of Detected Sound Paths

The number of paths is the number of valid direct and reflected sound paths from source to received that have been detected.

##### 4.2. Excess Attenuation

The excess attenuation expresses the relation of the sound pressure level at the receiver when compared to the sound pressure of the direct path between the source and the receiver. We calculate the excess attenuation using the following equation.

$$EA = 10 \log \frac{\int_{t_0}^{\infty} p_{total}^2(t) dt}{\int_{t_0}^{\infty} p_{direct}^2(t) dt} \quad (3)$$

where  $p_{total}$  is the total sound pressure at the receiver and  $p_{direct}$  is the pressure of the direct sound path arriving at the receiver.

##### 4.3. Reverberation Time

Reverberation time expresses the time required for sound energy to decay by 60 dB as described by the following equations [ISO08].

$$RT = \frac{60}{a} \quad (4)$$

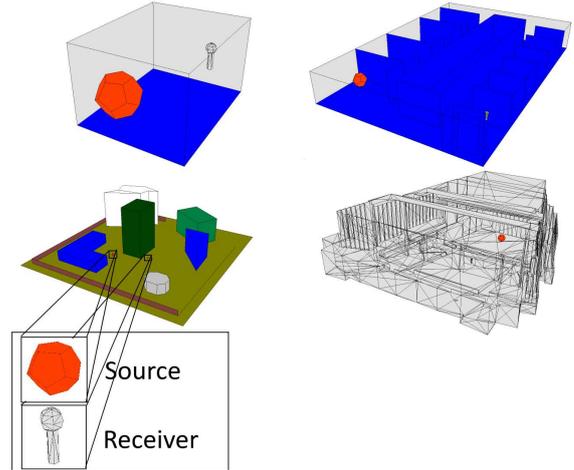
where  $a$  is the slope for the function

$$y = ax + b \quad (5)$$

calculated using the least squares method of the Schröder integration

$$E(t) = \int_t^{\infty} p(t)^2 dt \quad (6)$$

where  $p(t)$  is the relative sound pressure at time  $t$ .



**Figure 4:** The 3D geometries used for validation a. Shoe Box model (top-left) b. Multi-room indoor model (top-right) c. Outdoor model (bottom-left) d. Elmia Theater (bottom-right).

#### 4.4. Experimental Execution and Validation Models

We used our improved prioritized hybrid (IPH) algorithm and we compared its performance our to a typical hybrid algorithm (HT) without prioritization criteria, the first prioritized implementation (FPH) [CM16], a typical ray tracing (RT) algorithm and a ground truth solution. For the comparison we used four different 3D models, each with different characteristics, to cover as many different scenarios as possible. The models are the following.

1. Shoe-box model.
2. Multi-room indoor model.
3. Outdoor model.
4. Elmia theater.

In contrast with [CM16] where infinite material impedance was considered, we apply random absorption coefficient on each surface which is held constant between executions of different algorithms. This allows us to evaluate the addition of the minimum sound pressure criterion introduced in the IPH algorithm and how it compares with the FPH algorithm. The absorption coefficient is chosen randomly for values between 0 and 0.5 for each surface respectively and applied to all frequencies for the entire surface.

We chose a source receiver position for each model which would resemble a realistic scenario for that case. For example, in the Elmia theater we placed the source on the stage and the receiver in the middle of the audience. We ran each algorithm for 5 seconds. We implemented the code in C# and run the evaluation tests on a computer with an Intel Core i5-4200M Processor @ 2.50GHz.

## 5. Results and Discussion

### 5.1. Shoe Box

In Fig. 6, 5, 7 we can see the results of running the four algorithms on the Shoe-Box model. We can observe that FPH and IPH perform better than the other two on excess attenuation as they approximate

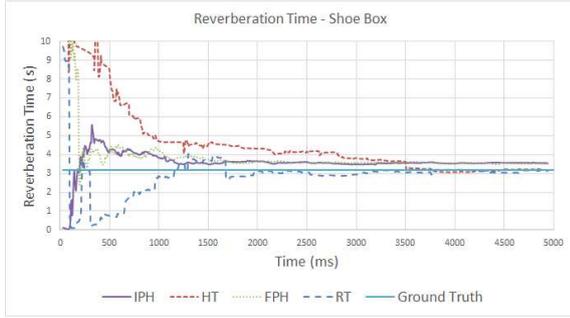


Figure 5: Reverberation for Shoe Box model.

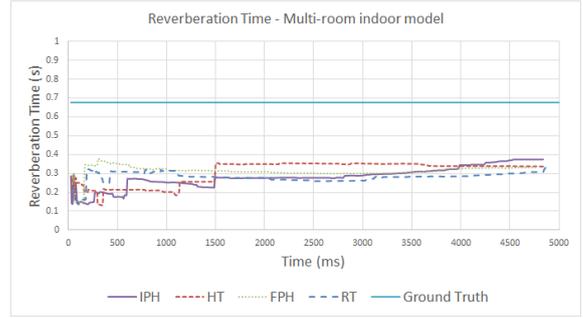


Figure 8: Excess attenuation for Multi-room indoor model.



Figure 6: Excess attenuation for Shoe Box model.

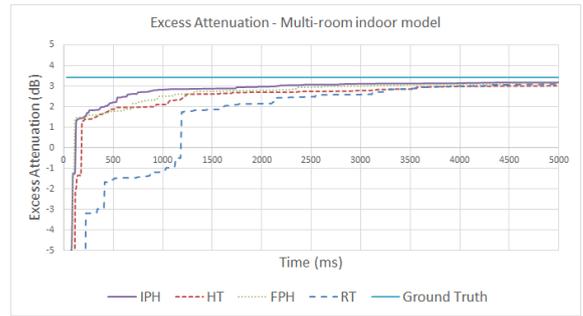


Figure 9: Reverberation time for Multi-room indoor model.

much faster the ground truth result. The difference varies from 2-5 dB for the largest part of the execution, a difference that can be considered perceptually important. IPH seems to perform slightly better than FPH.

In the reverberation time comparisons, FPH and IPH approach the ground truth solution much faster than the other two, even though a slight deviation is noted during the later parts of the execution. In the number of paths detected over time IPH performs better than the rest during the execution period.

### 5.2. Multi-room indoor model

Fig. 8, 9, 10 display the results of running the four algorithms on the Multi-room indoor model. We highlight again that FPH and IPH perform better than the other two on excess attenuation as they approximate faster the ground truth result. Again, IPH performs slightly better than FPH and hybrid tracing.

When it comes to reverberation time, none of the algorithms clearly outperforms the others since they all calculate a fluctuating reverberation time between 280 - 350 milliseconds for most of the duration of the execution. Comparing the number of paths detected IPH outperforms the rest at all stages of the execution.

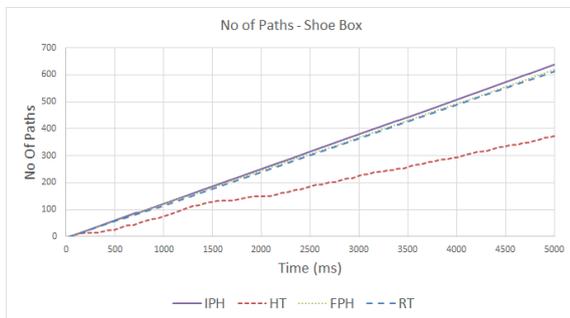


Figure 7: Paths for Shoe Box model.

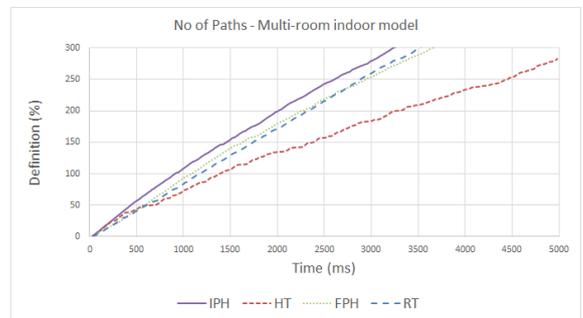


Figure 10: Paths for Multi-room indoor model.

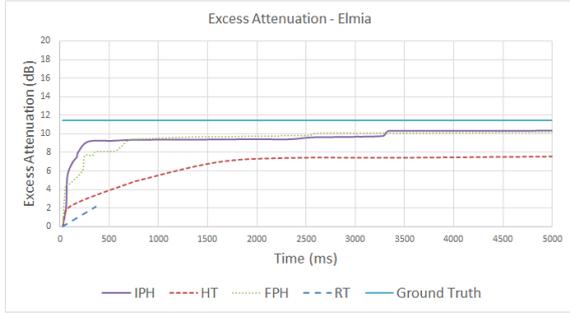


Figure 11: Excess attenuation for Elmia theater.

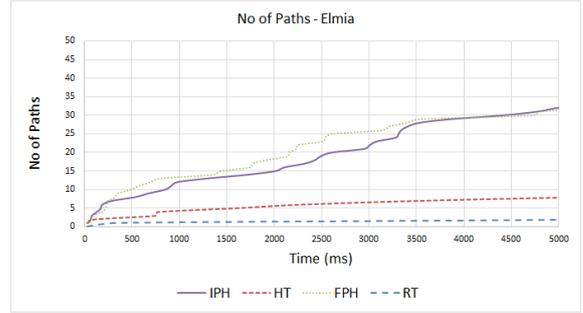


Figure 14: Excess attenuation for Outdoor model.

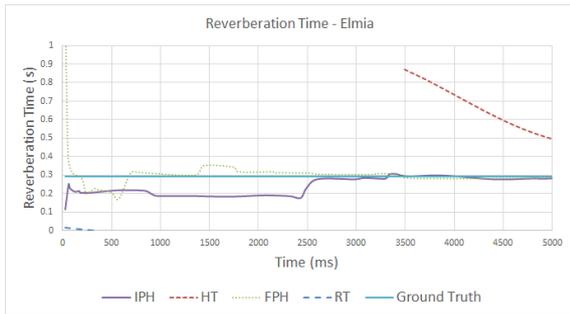


Figure 12: Reverberation time for Elmia theater.

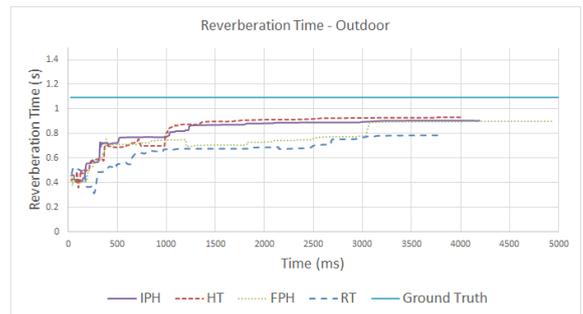


Figure 15: Reverberation time for Outdoor model.

### 5.3. Elmia Theater

The results for executions run for the Elmia theater are displayed in Fig. 8, 12, 13. Due to the fact that Elmia theater is a large model and large parts of its walls are composed by protrusions, few specular reflections exist when compared to scattered sound paths. Thus, the number of paths detected for all executions is small, a fact that does not allow safe conclusions. Nevertheless, we can see that FPH and IPH detect a much larger number of sound paths than the other two and this allows both algorithms to approach faster the ground truth solutions for excess attenuation and reverberation time. In Fig. 13, we can see that FPH outperforms IPH in the number of paths at the initial stages of the calculation. Both algorithms perform equally well in excess attenuation. In reverberation time FPH outperforms

IPH. Nevertheless, due to the small number of paths, the superiority might not be constant in subsequent executions. Ray tracing yielded to few paths for a calculation of a meaningful reverberation time and it is omitted from the reverberation time graph.

### 5.4. Outdoor model

Graphs in Fig. 14, 15, 16 contain the results for the outdoor model. In this case, despite the fact that the hybrid algorithm detects a much higher number of sound paths than the rest, in the case of excess attenuation IPH performs equally well. FPH and ray tracing under-perform in both the reverberation time and the number of paths.

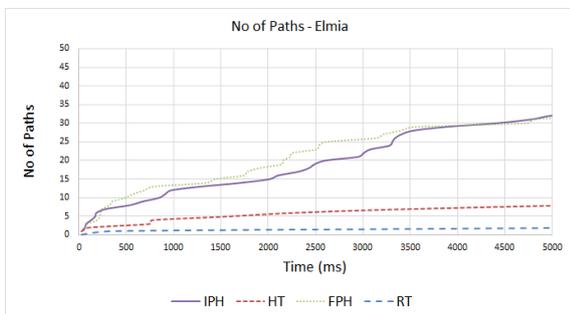


Figure 13: Paths for Elmia theater.

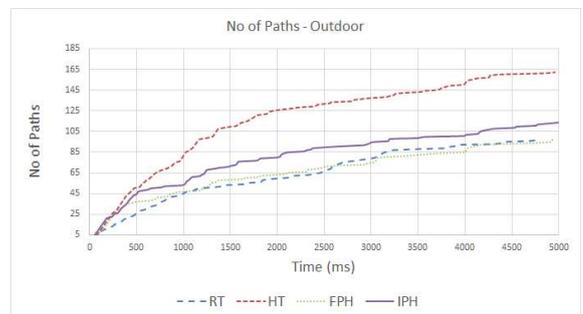


Figure 16: Paths for Outdoor model.

## 6. Conclusion and Future Work

We presented a method for improving the performance of geometrical acoustics algorithms used in sound rendering based on the concept of prioritization. We decomposed the tracing problem into a tree traversal problem and showed that GA algorithms are essentially tree traversals. By modifying a widely used hybrid tracing algorithm, we prioritize the traversal of the tree in such a way that more important nodes are validated earlier in the process. We achieve this by automatically adjusting tracing termination criteria during runtime instead of explicitly setting them beforehand. Simulation results of our method on four models with different characteristics show improvements in the calculated sound pressure and reverberation which are of perceptual significance for all the cases.

Beyond performance improvements our method has also the following benefits a) it removes the burden of setting termination criteria from the user. It has been shown that the selection of termination criteria can affect the performance of tracing methods [CM14b] [CE16]. Therefore, non optimum termination criteria can lead to performance deterioration. b) it does not replace previous methods but it can be used side by side. It can be used in parallel with any other GA method that implements a tree traversal. For example, it can enhance and improve any beam, ray or frustum tracing method.

Future work will focus on extending our algorithm beyond specular reflections to incorporate sound diffractions. Furthermore we will examine further attributes of the 3D model that might lead to better and more efficient prioritization. Lastly, we will perform perceptual evaluations with users.

## 7. Acknowledgments

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