# Improved Hybrid Algorithm for Real Time Sound Propagation using Intelligent Prioritization

Panagiotis Charalampous
Department of Multimedia and Graphics Arts
Cyprus University of Technology, Limassol, Cyprus,
Email:pc.charalampous@edu.cut.ac.cy

Despina Michael

Department of Multimedia and Graphics Arts
Cyprus University of Technology, Limassol, Cyprus,

Email:despina.michael@cut.ac.cy

Abstract—We present a hybrid algorithm which combines image source method, ray tracing and intelligent prioritization for faster calculation of impulse responses to be used in real time sound rendering. We use the image source method to calculate sound reflections from specular surfaces and we use a prioritized ray tracing algorithm for fast detection and evaluation of valid image sources from the tree of candidate image sources. Our algorithm delivers improved acoustical results in most cases when compared to a non prioritized version of it, a ray tracing algorithm and a best first image source algorithms, indicating that prioritization can deliver performance gains in real time sound rendering methods.

#### I. Introduction

Research in the field of computerized sound propagation calculation in three dimensional environments dates back to the 50s [1][2]. In the last two decades there is an increased interest in sound rendering for use in video games and virtual reality environments [3]. Immersive and realistic virtual environments require accurate representation of the sound scape in order to improve immersion, concentration, orientation, emotional response and overall satisfaction of the user [4] [5]. For this purpose realistic sound rendering needs to be incorporated within these environments.

Sound rendering is the process of generating impulse responses of three dimensional spaces and convolving them with anechoic recordings, to produce a realistic three dimensional sound [6]. It is the equivalent process to what graphics rendering is for light. Two major categories of techniques are used for the calculation of sound propagation. These are a) geometrical acoustics and b) numerical techniques. Geometrical acoustics is the approach of representing sound waves as rays. Numerical techniques are techniques which are based on the subdivision of the geometry into volume or surface elements and the solution of the relevant differential equations. In sound rendering and sound simulation in general, geometrical acoustics techniques are prevalent because of speed, even if they are considered not as accurate as numerical techniques. In geometrical acoustics, sound is described as a ray phenomenon. Sound rays propagation within an environment is simulated through a tracing procedure and valid sound rays are detected. The rays are then used for the estimation of a sound field at a given receiver position.

There are two major categories of tracing sound rays in a virtual environment. These are a) **deterministic tracing** and b) **stochastic tracing**. Deterministic tracing refers to the methods which will produce the same results after subsequent executions and stochastic tracing refers to the methods that use a Monte Carlo approach for the detection of sound paths, and the results might vary from execution to execution. For a detailed overview of these techniques see [3]. In this paper, we build on a hybrid technique proposed by Vorländer [7], which combines ray tracing and the image source method. We use this technique and we apply intelligent prioritization rules based on information gathered during the tracing. These rules result to intelligent choices of termination criteria, optimizing the use of limited time available in real time sound rendering scenarios.

#### II. RELATED WORK

Our work is based on the image source method, on ray tracing and on the use of intelligent algorithms for tree traversals. In the following sections we give a brief overview of the related work on these propagation techniques.

1) Image Sources: Image source methods compute virtual sources by considering each polygonal surface in the environment as a reflector. The location of the original source is mirrored in each reflector and new virtual sources are created. Virtual sources can then be used for the determination of reflection points, by finding the intersection of a line segment from the image source to the receiver. Then, the reflection points can be used for the construction of reflected sound paths. Finally, virtual sources can be recursively mirrored resulting to new virtual sources of higher order, therefore representing higher order reflections. The first computer code implementing the image source method was presented by Alen and Berkley [9] and it was intended for rectangular rooms. Borish extended it for arbitrary polyhedra [10] and Mechel has proposed an improved image source method which places criteria on the generation of the image sources [11]. Savioja et al. introduced a hybrid time-domain model for simulating room acoustics where direct sound and early reflections are obtained using the image source method and late reflections are modeled as exponentially decaying random noise functions [12]. Schröder accelerates the image source method by using binary space partitioning. Image source method is the basis of more sophisticated sound propagation techniques like beam and frustum tracing and it is also used in hybrid implementations, like Vorländer's hybrid method [7], which is used as the basis of our work.

2) Ray Tracing: Ray tracing is a technique widely used across many fields like radio propagation, computer graphics and acoustics. In acoustics, ray tracing is used as a technique

for generating an impulse response by tracing sound paths in a three dimensional environment and then calculating each path's contribution to the sound field. Ray tracing is the most popular geometrical acoustics technique and it is used by a number of commercial applications [13] and in interactive sound rendering.

In ray tracing, rays are emitted from a sound source in certain directions. There are many methods to obtain the ray direction. Some of them are the equal distribution of points on a sphere with a center the source point [15] and the use of statistical random distribution. [16]. Then the rays traced throughout the 3D space until they intersect a sphere which represents the receiver.

Ray tracing has been used in acoustics since 1958 [2]. The first work proposing a ray tracing algorithm for use in computers was published by Krökstad [15]. Then an improved algorithm for ray tracing for use in arbitrary room shapes was proposed by Kulowski [17]. Vorländer used a combination of ray tracing and image source model to calculate acoustical impulse responses for rooms [7]. A brief history of the use of ray tracing techniques for sound propagation has been published by Svensson [18]. Recent developments in ray tracing for sound rendering include the development of hybrid algorithms combining ray tracing with frustum tracing and methods for artificial reverb estimation [19], algorithms for the calculation of sound diffraction [20], ray tracing using multiview ray casting [21], ray tracing using acceleration structures [22] and ray tracing for higher order diffractions and diffused reflections [23].

3) Prioritization: To the authors knowledge, not much research has been done on prioritization for geometrical sound propagation. One work dealing with this subject is Min and Funkhouser's priority based beam tracing [24] and also a study on tree traversals for real time sound propagation [25].

#### III. OUR ALGORITHM

# A. Vorländer's Hybrid Method

Vorländer presented a method which combines the advantages of both ray tracing and the image source algorithm [7]. In brief the method goes as follows a) a ray is emitted from the source and propagated through space b) as soon as the ray hits a surface, the surface's image source is generated and recorded c) The ray is reflected from the surface d) steps b and c are repeated recursively until the ray intersects a predefined receiver sphere, as it would in a typical ray tracing algorithm. e) the ray is neglected and the image sources recorder during the ray tracing are evaluated for their validity. A valid image source is an image source which actually produces a specular sound reflection. On the other hand, non valid image sources are image sources for which the reflection point is outside their surface area. The valid image sources are used for the estimation of the impulse response.

# B. Prioritization of the Hybrid Method

The hybrid method presented above improves significantly the performance of image source algorithm but when it comes to real time sound rendering, it has an important drawback. Vorländer's tracing process terminates when a) the ray intersects the receiver sphere or b) when it reaches a certain

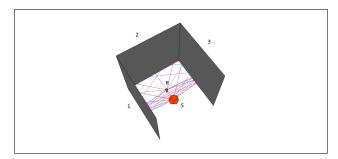


Fig. 1. Valid reflection paths up to the 3rd order in a source receiver configuration with three surfaces.

energy level or c) a predefined traveling distance is covered. In room-like enclosures, for which this algorithm was designed, this termination criterion works well, because after some bounces on the wall, the ray eventually intersects with the receiver. But these criteria do not work well in other types of environments, indoor configurations with many rooms and outdoor configurations. The reason is because ravs will be probably shot in directions from were it is not easy to return to the receiver and until they meet the termination criteria and the tracing is interrupted, a lot of computation time is wasted. Most modern ray tracing implementations use one of the following termination criteria a) a limit in sound reflections allowed per path, meaning that the propagation of a sound ray is terminated after a certain number of bounces has occurred and the receiver has not been reached [19] [21] [23] b) a minimum energy criterion where the ray propagation is terminated after its energy falls under a certain level [1] [26] c) a maximum distance criterion where the ray propagation is terminated after the ray surpasses a predefined traveling distance [22]. The termination criteria are usually set arbitrarily, e.g. ten orders of sound reflections or a maximum distance of 1000, without any further discussion or based on a guessed perceptual importance e.g sound paths that loose 60 dB are probably not affecting significantly the sound field. Henceforth, we will explain why this way of setting termination criteria affects significantly the performance of a ray tracing algorithm when used for real time sound rendering.

We improve the above algorithm by applying a more intelligent termination criterion. Our improvement is based on the following grounds. At first, all tracing algorithms based on specular sound paths are variations of tree traversal algorithms [25]. Image source algorithm is the most typical tree traversal algorithm. Variations of the image source algorithm like beam tracing and frustum tracing are all tree traversal algorithms that base their speed improvements on the concept of pruning certain nodes of the tree, thus decreasing substantially the nodes needed to be examined. In a similar way, stochastic approaches like ray and particle tracing use a Monte Carlo approach in visiting tree nodes, incorporating an inherent visibility pruning of the tree. The above mentioned methods achieve acceleration in execution by pruning parts of the tree that cannot generate any valid sound reflections and by speeding up the tree traversal process.

We enhance the tree traversal process by improving the way we evaluate tree nodes for potential specular reflections. Our enhancements are based on the following observations.

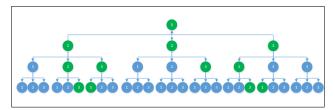


Fig. 2. Image sources tree of Fig. 1 configuration.

Higher trees have lower valid image sources density. It is generally observed that each level of the image source tree has a lower density of valid sources to total sources than the previous ones. This can be expressed by the following relationship.

$$P(V|O) < P(V|O-1)$$

Where V indicates that an image source is valid and O the image source order. The above expression can be phrased as the probability of an image of order O to be valid is less than the probability of an image of order O-1. Fig. 1 and Fig. 2 show an example of such a case. Fig. 1 shows a simple configuration with a source, a receiver and three surfaces. We have detected the specular sound reflections up to the third order using the image source algorithm and displayed them in a 3D viewport. Fig. 2 shows the image source tree of the specific configuration. In green, you can see the valid image sources. Within the node, the image source surface is displayed. The surfaces are represented by a number and are associated with Fig. 1. By examining the nodes of the tree level by level, we can observe that the ratio of valid to total images is 3/3 for the first level, 4/9 for the second level and 4/27 for the forth level. This behavior holds true for any 3D model. More information about this behavior of the image source method can be found in [7] [11]

• Higher probability of a valid parent node than the average density of valid sources in the parent level. Another fact that can be observed in the image source tree is that the probability of an image source having a valid parent image source is higher than the average density of valid sources in the parent level [7] [11] [27]. This can be expressed by the following relationship.

$$P(V|P_V) > P(V|P_{NV})$$

Where V indicates that an image source is valid,  $P_V$  a valid parent source and  $P_NV$  a non valid parent source. The above expression can be phrased as **The probability of an image of with a valid parent source to be valid is less than the probability of an image with a non valid parent source.** 

In Fig. 2, this can be demonstrated taking the third level of image sources as an example. In this level 3 in 4 valid image sources have a valid parent source. When compared to the 4 in 9 valid image sources in the second level, the probability of an image source having a valid parent is higher than the probability

of the parent being valid or not. A intuitive physical explanation of this fact is the following. It is more probable that surfaces that reflect sound back to the receiver to also reflect the sound to surfaces that also send the sound wave back to the receiver, than surfaces that do not reflect any sound to a receiver.

The hybrid ray tracing/image procedure described earlier [7], is practically equivalent with the traversal of a random path in the tree, up to a certain depth of termination. Using the above two observations, we made the following conjecture. If we could prioritize the rays to give preference to the higher level nodes of the tree or paths that expand from valid images rather than from non valid ones, we would be able to generate more valid sound reflection paths within the same timespan of execution. As a result, in our algorithm we introduce two different termination criteria. These are the following.

- Maximum Non Valid Images. The first termination criterion is related with the fact that it is more probable that a valid image source has a valid child source than a non valid one. Based on this, rays reaching a tree node with a non valid image source have less probabilities, to reach a valid child node than rays that reach a valid tree node. Therefore, we terminate a ray propagation as soon as a maximum number of non valid images has been reached within the tree path traversed.
- Maximum Tree Depth. The second termination criterion is related with the fact that higher levels of the tree have a higher density of valid image sources. This means that if more rays explore the higher levels of the tree rather than the lower, there is an increased possibility of detecting valid image sources. Moreover, lower order images are usually stronger than the higher order ones, thus contributing more in the final sound field. Therefore we set a maximum order criterion.

A ray is terminated if it exceeds both criteria. In order to achieve the prioritization of sound rays, the above criteria are progressively increased. This way we achieve to send rays in those sections of the tree that have the highest probability of containing valid image sources. More precisely the steps of our algorithm are the following.

- We initialize the termination criteria setting 0 for Maximum Non Valid Images and 1 for Maximum Tree Depth.
- We emit a ray using a vector from the source point to the a statistical random distribution over a sphere.
- 3) We check which surface is intersected by the ray and we evaluate the respective image source. If the image source is valid, we keep track of the sound path.
- 4) We check if both termination criteria apply. If they apply, we terminate the tracing of the ray. Else we continue tracing the bounces of the ray until both termination criteria are satisfied.
- As soon as the termination criteria are met, we trace a new ray.
- 6) After a number of consecutive rays with no valid images, we increase the termination criteria by 1.

The number of failed consecutive rays is variable and is equal to the number of reflecting surfaces with an environment e.g. if the 3D model contains 50 reflecting surfaces, the termination criteria increase after 50 consecutive rays have failed to produce valid sound paths. The variable number of failed rays was chosen in favor of a static one to ensure a statistically better coverage of wider image source trees.

# IV. RESULTS AND DISCUSSION

We used our prioritized hybrid (PH) algorithm and we compared its performance to a typical hybrid algorithm without prioritization criteria, a typical ray tracing (RT) algorithm and a best first image source approach presented in a previous work of the authors [25]. For the comparison we used five different 3D models, each with different characteristics, to cover as more different scenarios as possible. The models are the following.

- Shoe-box model. We used a shoe-box model which is the typical scenario in room acoustics calculations.
- 2) **Elmia theater**. The Elmia theater was used in the 2nd Round Robin for room acoustics [28].
- Multi-room indoor model. We used an indoor environment consisting of many interconnecting rooms, resembling a typical scenario found in virtual reality and video games environments.
- Choir rehearsal room. We used the choir rehearsal room used in [25] for comparison with previous methods
- 5) **Outdoor model**. We used the outdoor model used in [25] for comparison with previous methods.

We chose a source receiver position for each model which would resemble a realistic scenario for that case. For example, in the Elmia theater we placed the source on the stage and the receiver in the middle of the audience. Due to the stochastic nature of the hybrid methods, in order to avoid the possibility of a non representative calculation, we ran the algorithms for 10 consecutive times and picked the run which produced more sound paths. We implemented the code in C# and run the evaluation tests on a computer with an Intel Core i5-4200M Processor @ 2.50GHz.

We compared the algorithms based on the following results a) Excess attenuation b) Reverberation time c) Strength accumulation. Following on, we give a brief description of each parameter. We omit the calculation of sound attenuation from surface impedance and atmospheric absorption. This allows us to omit the calculation of ground truth results since without any sound attenuation in closed rooms, the energy is sustained infinitely and the results are theoretically infinite. Therefore, this simplifies the evaluation of the results since the highest is always closer to the ground truth.

1) **Excess attenuation**. The excess attenuation expresses the relation of the sound pressure level at the receiver when compared to the sound pressure of the direct path between the source and the receiver. The excess attenuation is calculated using the following equation.

$$EA = 10\log\frac{\int_{t_0}^{\infty}p^2(t)dt}{\int_{t_0}^{\infty}p_{direct}^2(t)dt}$$

where  $p_{total}$  is the total sound pressure at the receiver and  $p_{direct}$  is the pressure of the direct sound path arriving at the receiver.

- Reverberation Time. Reverberation time expresses the time required for sound energy to decay by 60 dB. The detailed calculation of the reverberation time can be found here [29].
- 3) **Strength**. Sound strength represents the energy that exists in the early parts of the signal and expresses the perception of the strength of the sound. It is calculated with the following equation.

$$G = 10 \log \frac{\int_{t_0}^{\infty} p^2(t)dt}{\int_{t_0}^{\infty} p_{10}^2(t)dt}$$

where p is the sound pressure arriving at the receiver at time t and  $p_{10}$  is the sound pressure arriving at the receiver from a sound source at 10 meters distance.

We compare the above parameters in terms of accumulation over time. We do this on purpose because we want to show how each algorithm behaves over time. A fundamental difference between real-time simulations and static simulations is that the execution of an algorithm might be interrupted at any time based on the available resources. An accumulation based examination of the result does not only focus on the final calculation result but also on what the result would be if the execution was interrupted at any time of the execution. Also we focus on the relative difference between these algorithms and not on absolute computation time. Absolute execution time is highly depended on implementation details, like the programming language used, programming techniques and the system's resources, something which is out of the scope of the current study.

## A. Shoe-Box

The shoe-box model (Fig. 3) is a 3m\*4m\*5m room, commonly used in acoustics simulations. It has 12 triangles. In this case, we can observe that prioritized hybrid tracing (PH) outperforms other algorithms in terms of sound pressure level (SPL) accumulation over time while in the case of reverberation time the best first approach surpasses the others in performance after some time of execution. Both non prioritized algorithms, hybrid and ray tracing, perform worse than the prioritized ones. Differences in SPL are 2 dB minimum and in RT 0.5 seconds.

### B. Elmia Theater

The Elmia theater (Fig. 4) is a theater model that has been used in the second round robin on room acoustical computer simulation [28]. It consists of 1908 triangles which result to a very wide image tree when compared to the typical shoe box room. Observing the results of the runs using the Elmia theater, we can see that all algorithms perform very poorly compared to PH. The reverberation time results for all algorithms except PH are missing from Fig. 4, since they failed detect enough sound paths for the calculation of the reverberation time.

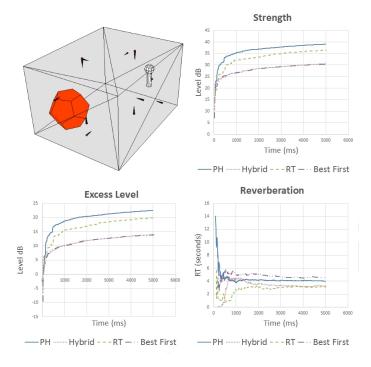


Fig. 3. Shoe-Box model and results.

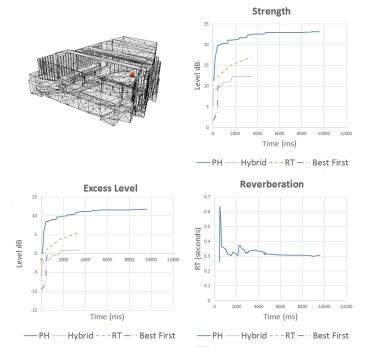


Fig. 4. Elmia theater model and results.

# C. Multi-room indoor model

The multi-room indoor model (Fig. 5) is an indoor environment consisting of many interconnecting rooms, resembling a typical scenario found in virtual reality and video games environments. It consists of 88 triangles. In this model, PH and RT behave almost equally in terms of SPL accumulation, outperforming the rest, even though PH is slightly ahead at

any point of execution. When compared based on reverberation time, after the first unstable stages of the execution, PH clearly outperforms the rest.

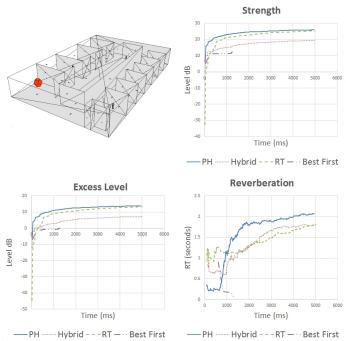


Fig. 5. Multi-room indoor model and results.

#### D. Choir rehearsal room

The choir rehearsal room (Fig. 6) is a medium sided room and is one of the models we have used in previous work [25]. 58 triangles are present in this model. For SPL parameters, the results are similar to the shoe box room but for reverberation time, PH outperforms significantly all the rest by at least 1s.

## E. Outdoor model

The outdoor model is a model of 94 triangles(Fig. 7), that is not completely bounded by surfaces. In this case, an almost identical behavior is observed in SPL accumulation for PH and ray tracing, outperforming the other two algorithms, while in reverberation time there is no algorithm which outperforms constantly all the other for the entire period of execution.

# V. CONCLUSIONS AND FUTURE WORK

Real-time sound simulation is fundamentally different than static off-line simulation. This is because in real-time simulation resources are limited, unpredictable and unstable. Therefore, algorithms designed for real time simulation should be designed with different criteria in mind. These criteria should be a) **User independence**. We have shown above how typical hybrid algorithm changes its behavior based on different user defined termination criteria. Also, the same termination criteria behave differently in different 3D models. A real time algorithm's performance should not change based on such user intervention. b) **Constant result improvement over time**. Real time execution can be interrupted at any time. A real

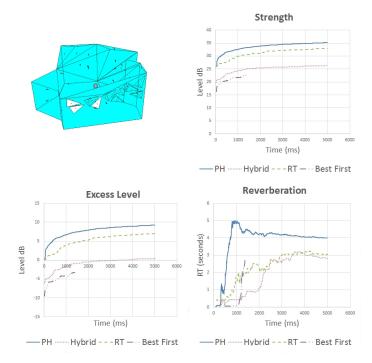


Fig. 6. Choir rehearsal room model and results.

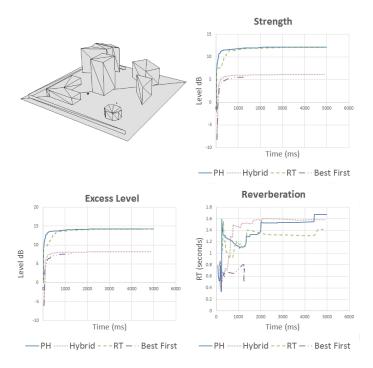


Fig. 7. Outdoor model and results.

time algorithm should give the best possible result at the given interruption time.

In this study, we have compared four algorithms a) a hybrid image source/ray tracing implementation with user defined termination criteria (Hybrid) b) a ray tracing algorithm with user defined termination criteria (RT) c) a hybrid implementation which prioritize ray tracing and tree traversal by progressively

relaxing the termination criteria (PH) and d) a best first image source tree traversal using prioritization criteria (Best First). We conclude that our proposed hybrid algorithm with its automated criteria setting method outperforms all the other algorithms in most cases investigated and yields results that fulfill better the quality criteria set above for a real-time sound propagation algorithm when compared to implementations using predefined termination criteria. Also, the improvements achieved using prioritization are of perceptual significance. Hence, we conclude that the concept of prioritizing the execution of sound propagation algorithms deserves more attention and deeper investigation in the future.

A limitation of our algorithm is that it still contains some arbitrary set values, like the number of consecutive failed rays. This might affect performance in a similar way to arbitrary set termination criteria. Future plans include further reduction in the arbitrariness of the algorithm, the examination of prioritization using other algorithms too and also the evaluation of more sophisticated termination criteria.

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